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ABSTRACT

Ten review lessons in basic mathematics are presented in this workbook designed for electrical apprentices and vocational students. The lesson topics are numbers (whole, positive, negative), fractions, decimal fractions, finding common denominators, reciprocals, percentage, powers and square roots, ratio and proportion, the circle and its use of formulas, and negative exponents. Each lesson generally consists of an information sheet, which explains the mathematical operation and gives examples of some problems, and worksheets of problems to be solved by the student. (JH)

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BASIC MATHEMATICS REVIEW

ED 160876

INFORMATION SHEETS AND WORKSHEETS
FOR
APPRENTICE AND VOCATIONAL STUDENTS

Prepared by
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and
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August 1971
Revised to meet 1975 Electrical Code

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PROGRAM DEVELOPMENT
DEPARTMENT OF COMMUNITY COLLEGES
RALEIGH, NORTH CAROLINA

CE 018 351

FOREWORD

The apprenticeship programs offered in North Carolina are important phases of the total educational program for industry in the state. The program of education for modern apprenticeship includes two distinct phases. The first phase is "on-the-job training." The student is employed by a firm in which he performs duties specifically assigned to enhance his learning in his field of study. These experiences are planned and supervised so that the individuals will progress through the apprenticeship phase to that of the journeyman. The second phase of the program of education is termed "related instruction." In this phase the apprentice is provided the technical and theoretical foundation subjects that help him to learn the trade thoroughly and accurately. This portion is usually taught through evening school, but in some cases may be done through correspondence courses.

The North Carolina Department of Community Colleges, through the Program Development Section, has the responsibility for developing and making available the instructional materials that provide the learner with a sound educational program which is effective and appropriate to the method in which the materials are being utilized. The department has available these materials for the students, as well as support material for the instructor who teaches or who acts as counselor in conducting the related classes.

Roger G. Worthington
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Department of Community Colleges

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BASIC MATHEMATICS REVIEW

INTRODUCTION

A knowledge of elementary mathematics is most essential to an understanding of the principles of electricity. Thus, it may be said that mathematics is used as a tool in the study of electricity.

This review of Basic Mathematics, the information sheets and worksheets have been prepared to help you attain the knowledge you seek. A review of the fundamental operations using both whole numbers and fractions is necessary before attempting the more advanced problems dealing with electricity.

The electrical apprentice is expected to make correct measurements of conductors, conduit, and other materials used on the job. Later, he will learn how to calculate the maximum number of conductors that can be pulled into certain sizes of conduit. It is important to make use of the basic mathematics that you know in studying about electricity and those things you don't know.

The Basic Mathematics Review Unit consists of Information Sheets that explain the operations and show examples of elementary problems. Worksheets are included to give the student practice in reviewing these basic problems. Any program may use this material.

Students that review the Basic Mathematics Unit should also refer to the Basic Mathematics textbook used in the program. The textbook will give more examples and problems that are related to electricity.

BASIC MATHEMATICS REVIEW

INFORMATION SHEET

Signs and Their Meaning

X or ·	Multiplied by	□	Square
÷	Divided by	◎	Circle
/	Divided by	°	Degrees
+	Plus (addition)	'	Minutes or feet
+	Positive	"	Seconds or inches
-	Minus (Subtraction)	( )	Parentheses
-	Negative	{ }	Braces
$\sqrt{}$	Square root	[ ]	Brackets
:	is to :: as : is to	2^2	Two squared
>	Greater than	2^3	Two cubed
<	Less than	π	3.1416 (pi)
∠	Angle	$\frac{\pi}{4}$	.7854
=	Equals	%	Percent

INFORMATION SHEET Lesson: I

I. Numbers

A. Whole numbers

Numbers play a very important part in our everyday affairs. We need to know how to correctly read and write numbers. We should thoroughly understand the fundamental operations, that is, how to add, subtract, multiply, and divide numbers.

It has been found that most of the errors made in the more complicated math problems are made in the fundamental operations. It is necessary for the student to review the basic mathematics operations before attempting to solve problems involving alternating current. Certainly, mathematics is used as a tool in the study of electricity.

B. Positive and negative numbers

In arithmetic, the plus (+) and minus (-) signs are signs of operation; that is, they tell us to add or subtract a number. We now come to a new use of these signs, a use which shows the quality of a number.

For example: If we are told the temperature is ten degrees, we really cannot be sure whether the temperature is ten degrees above or ten degrees below zero. If ten degrees below zero is meant, however, we have only to qualify the number by a minus sign. The minus sign when so used does not mean subtract, but simply shows to which side of the zero mark the number applies.

Thus, we can picture positive and negative numbers as shown on the scale below:

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

It is seen, then, that the plus or minus sign indicates in which direction a number applies. When the positive direction is meant, the plus sign is usually omitted, but when the negative direction is meant, the minus sign is always used. Positive and negative numbers, when considered in conjunction with zero, are called algebraic numbers.

BASIC MATHEMATICS REVIEW

NAME_____ Lesson: I

DATE_____ Worksheet: IA

Whole Numbers

I. Whole Numbers

A. Review of addition, subtraction, multiplication, and division

PROBLEMS

1. Add the following:

(a) 472 + 26 (b) 529 + 358 (c) 1741 + 1902 (d) 420 + 945 + 441 + 584 + 715

2. Subtract the following:

(a) 7418 − 6356 (b) 5747 − 1856 (c) 7904 − 5738 (d) 4408 − 994

3. Multiply the following:

(a) 719 × 4 (b) 853 × 74 (c) $4.08 × 58 (d) $5.76 × .39

4. Divide the following:

(a) $4\overline{)6404}$ (b) $5\overline{)8525}$ (c) $62\overline{)3596}$ (d) $48\overline{)127,924}$

BASIC MATHEMATICS REVIEW

NAME_____ Lesson: I

DATE_____ Worksheet: IB

(+) and (-) Numbers

B. Positive and negative numbers

Add the signed numbers indicated in each problem.

1. (+3) + (+9)

2. (-6) + (-5)

3. (+8) + (-4)

4. -26
+12

5. +36
-14

6. -10.5
+12.4

7. (+5/8) + (-1/4)

8. -16 7/8
+ 8 3/4

9. -15 3/4
+28 1/2

10. +8 -2 -9 +6

11. -6 -3 +11 +4 -5

12. -16 +4.8 +3.4

Multiply as indicated.

13. (-3) by (-4)

14. (-3) by (6)

15. (5R) by (-2)

16. (-2R) by (-5)

17. (-0.2) by (5R)

18. (-1/2) by (-20)

19. (-16I) by (-0.2)

20. (-0.6) by (8.2R)

INFORMATION SHEET Lesson: II

II. Fractions

A. Common fractions

- If a whole number is divided into parts, this division is expressed as a fraction.

  For example: $\frac{2}{5}$ $\frac{3}{4}$ $\frac{7}{8}$ $\frac{9}{27}$ $\frac{3}{16}$ are all fractions. Each of them is less than one whole number. The figure under the line is known as the denominator, and the figure above the line is called the numerator.

- If the numerator is less than the denominator, the fraction is called a proper fraction. 3/4 is a proper fraction.

- If the numerator is greater than the denominator, the fraction is called an improper fraction. 8/7 is an improper fraction.

- A proper fraction should be reduced to its lowest terms.

  For example: 24/60 equals 4/10 equals 2/5

- An improper fraction should be expressed as a mixed number.

  For example: 5/2 equals 2 1/2.

- A mixed number such as 2 1/2 may be changed to an improper fraction (5/2) to aid one in adding, subtracting, multiplying, or dividing fractions.

B. Addition of fractions

If the fractions are equivalent, (i.e. having the same denominator) it is only necessary to add the numerators, the denominators remaining the same.

Example: $\frac{2}{5} + \frac{3}{5} + \frac{1}{5} + \frac{4}{5} = \frac{10}{5} = 2$

If the fractions are not alike, it is first necessary to reduce them to equivalent fractions, and then add the numerators. Such as:

$$\frac{1}{4} + \frac{1}{3} + \frac{5}{8} + \frac{7}{16} = \frac{12}{48} + \frac{16}{48} + \frac{30}{48} + \frac{21}{48} = \frac{79}{48} = 1\frac{31}{48}$$

When adding mixed numbers, reduce to improper fractions and proceed as above.

C. Subtraction of fractions

- If the fractions are equivalent, it is only necessary to subtract the numerators.

  For example: $\frac{7}{8} - \frac{5}{8} = \frac{2}{8} = \frac{1}{4}$

- If the fractions are not alike, reduce them to equivalent fractions and subtract the smaller numerator (the subtrahend) from the larger numerator (the minuend).

  The result is the remainder. Such as:

$$\frac{7}{8} - \frac{7}{32} = \frac{28}{32} - \frac{7}{32} = \frac{21}{32}$$

D. Multiplication of fractions

- To multiply a fraction by a whole number, multiply the numerator by the whole number. This will be the numerator of your answer. The denominator remains unchanged.

  For example: $\frac{5}{8} \times 7 = \frac{5}{8} \times \frac{7}{1} = \frac{35}{8} = 4\frac{3}{8}$

  Multiplicand x multiplier = product

- To multiply a fraction by another fraction, multiply through the numerators. This will give the numerator of your answer. Then multiply through the denominators. This will give the denominator of your answer. Cancel where possible. Reduce mixed numbers to improper fractions.

  For example: $\frac{7}{8} \times \frac{5}{25} = \frac{35}{88}$

  $$\frac{1}{3} \times \frac{2}{5} \times \frac{15}{2} \times \frac{16}{20} = \frac{1}{5} \times \frac{3}{4} \times \frac{3}{20} = \frac{3}{200}$$

E. Division of fractions

In division, we set up our problem as follows:

dividend ÷ divisor = quotient

$\frac{17}{32} \div \frac{2}{5} = ?$

In dividing, the rule is to invert the divisor and then multiply. So we put our problem thus:

$\frac{17}{32} \times \frac{5}{2} = \frac{85}{64} = 1\frac{21}{64}$

Reduce all mixed numbers to improper fractions before proceeding as above.

For example: $6\frac{2}{3} : 2\frac{3}{4} = \frac{20}{3} \div \frac{11}{4}$

Inverting the divisor and multiplying, we have

$\frac{20}{3} \times \frac{4}{11} = \frac{80}{33} = 2\frac{14}{33}$

BASIC MATHEMATICS REVIEW

NAME_____ Lesson: II

DATE_____ Worksheet: II

Fractions

II. Fractions

1. Reduce to lowest terms:

   (a) 4/8 =

   (b) 4/16 =

   (c) 14/32 =

   (d) 60/100 =

2. Reduce mixed numbers to improper fractions:

   (a) 5 3/8 =

   (b) 3 1/3 =

   (c) 10 7/8 =

   (d) 2 1/12 =

3. Changing improper fractions to mixed numbers:

   (a) 9/2

   (b) 20/2

   (c) 59/8

   (d) 43/10

4. Multiplication of fractions:

   (a) 8 x 1/2 =

   (b) 2/5 x 1/3 =

   (c) 3/8 x 16 =

   (d) 1/4 x 3/8 =

5. Multiplication of mixed numbers:

   (a) 1 7/9 x 3/8 =

   (b) 3 3/5 x 2 1/4 =

6. How many nipples, 1 1/2 inches long can be made from a piece of conduit 24 inches long? _____.

INFORMATION SHEET Lesson: III

III. Decimal fractions

Many students find that decimal fractions are much easier to learn and to work with than common fractions. In writing the figures for a dime, we have $0.10 which is a decimal fraction. We do not use the common fraction 1/10 of a dollar. In the study of electricity, decimal fractions are used constantly, and also the powers of 10 will be used. Therefore, we must be sure that we know how to handle problems involving decimal fractions.

Examples: We write the diameter of #10 RH wire as .242' not 242/1000. If a lamp uses eighty-three one hundredths ampere; we write it .83 ampere not 83/100 ampere.

If the denominator of a fraction is 10, 100, 1000 etc., we may omit the denominator entirely, replacing it by a decimal point in the numerator, so that there are as many figures to the right of the decimal point as there are zeros in the denominator.

For example: 3/100 = 0.3, 41/1000 = 0.041, 3287/10000 = 0.3287

Fractions, written without a denominator, but with a decimal point, are called decimal fractions, or decimals, for short.

Every time we move our decimal point one place to the right, we increase the value of that number by ten times.

For example: 4.86 x 10 = 48.6
4.86 x 100 = 486.
4.86 x 1000 = 4860.

Every time we move our decimal point one place to the left, we decrease the value of that number by ten times.

For example: 4.86 10 = .486
4.86 100 = .0486
4.86 1000 = .00486

In reading decimals, it is common practice among engineers to use the word "point" instead of "decimal" or "decimal point." Thus, 54.286 is read "five four point two eight six."

1. Addition of decimals

Write the numbers so that the decimal points are under each other and proceed as with whole numbers. Put the decimal point in the answer under the other decimal points.

Examples:

152.31	32.6
465.13	514.2
394.26	2.3
283.78	7413.4
1295.48	7962.5

2. Subtraction of decimals

Write the numbers so that the decimal points are under each other, and proceed as with whole numbers. Keep in mind that zeros may be added to the right of a decimal point without changing its value. Thus, 0.2 = 0.20. Therefore, to subtract 13.4256 from 26.7, add zeros to the right of the seven, thus:

$$\begin{array}{r} 26.7000 \\ 13.4256 \\ \hline 13.2744 \end{array}$$

3. Multiplication of decimals

Regard the decimals as whole numbers, multiply, and point off in the product as many decimal places as the sum of the decimals in the multiplicand and the multiplier.

Examples:

0.68	0.00343
x 0.23	x 0.00026
204	2058
136	686
.1564	.0000008918

4. Division of decimals

In division, it is essential to remember that the DIVIDEND ÷ DIVISOR = QUOTIENT.

or $\underline{\text{DIVISOR}}\ )\ \overline{\text{DIVIDEND}}$ (QUOTIENT)

Insofar as the actual dividing is concerned, decimals are treated as whole numbers.

If we remember to move our decimal point in the divisor the necessary number of places to the right to make it a whole number, and then move the same number of places in the dividend, we will have no difficulty in placing the decimal point in our answer, the quotient. It will be directly above the decimal point in the dividend.

For example: .24) 6.53664 would be set up thus:

```
      27.236
24. ) 653.664
      48
      173
      168
       56
       48
        86
        72
        144
        144
```

If the dividend is a whole number, and the divisor a decimal, we will still follow the same rule by adding the necessary number of zeros to the dividend to correspond with the number of places we move to the right in the divisor to make it a whole number.

For example: .372) 4323. would be set up as 372.) 4323000.

and proceed as before.

BASIC MATHEMATICS REVIEW

Lesson: III

NAME_____ Worksheet: III

DATE_____ Decimal Fractions

III. Decimal Fractions

1. Change fractions to decimal fractions:

   (a) 6/10 (c) 15/1000

   (b) 50/100 (d) 5/100

2. Arrange in order starting with largest:

   (a) 0.07, 0.005, 0.15, 0.3 _____

   (b) 0.18, 0.002, 0.07, 0.0010 _____

   (c) 0.294, 0.790, 0.02 _____

3. Change mixed numbers to decimals:

   (a) 2 5/10 (c) 1 18/100

   (b) 3 35/100 (d) 65 14/1000

4. Change fractions to decimals:

   (a) 1/5 (c) 25/32

   (b) 1/3

5. Work following as directed:

   (a) 8.23
5.90
.436
+ .870

   (b) 1.63
- 0.81

   (c) 0.005
x 28

(over)

BASIC MATHEMATICS REVIEW

NAME_____

DATE_____

Lesson: III

Worksheet: III

Decimal Fractions
(cont'd)

(d)
$$\begin{array}{r} 0.078 \\ 6.810 \\ 0.040 \\ +\ .719 \\ \hline \end{array}$$

(e) $9.6 \div 0.4$

(f)
$$\begin{array}{r} 0.613 \\ -0.045 \\ \hline \end{array}$$

(g) $100 \times .037 =$

(h) $80.8 \div 0.08 =$

INFORMATION SHEET Lesson: IV

IV. How to find common denominators

If a number is divisible by another number, it is a multiple of that number. If that number is exactly divisible by more than one number, it is a common multiple of all those numbers.

For example: 48 is a common multiple of 2, 3, 4, 6, 8, 12, 16, and 24.

Two or more numbers may be common multiples of several smaller numbers. Thus, 48 and 36 are both common multiples of 2, 3, 4, 6, and 12, but 36 is the least common multiple (L.C.M.) of these numbers. Therefore, to find the L.C.M. of a series of numbers, we must find a number into which all of these numbers will divide. The process is to find the smallest number that will divide two or more of the numbers, then to find another number that will divide two or more of the remaining numbers, bringing down the numbers which cannot be divided, after each operation, continuing as long as possible. The least common multiple (L.C.M.) is, then, the product of all the divisors.

For example: Find the L.C.M. of 3, 8, 9, and 32.

3	3	-8	-9	-32
8	1	-8	-3	-32
3	1	-1	-3	- 4
4	1	-1	-1	- 4
	1	-1	-1	- 1

Multiplying all the divisors, we have 3x8x3x4 = 288, which is the least common multiple. The same method may be used for finding the least common denominator of a number of fractions which we desire to change to equivalent fractions in order that we may add them all together.

The reciprocal method of finding total resistance in a parallel circuit where we have several paths, each of which has a different resistance, is a typical case of how the above method may be used in the solving of electrical problems. Suppose we have four parallel paths with a resistance of 40 ohms in path #1, 60 ohms in path #2, 30 ohms in path #3, and 20 ohms in path #4.

We set up our problem thus:

$$\frac{1}{40} + \frac{1}{60} + \frac{1}{30} + \frac{1}{20} = \frac{3}{120} + \frac{2}{120} + \frac{4}{120} + \frac{6}{120} = \frac{15}{120}$$

Since we desire the reciprocal of this total we turn the fraction over and divide (120/15) or 8 ohms as our total resistance.

The greatest common divisor.

The greatest common divisor or G.C.D. is the largest number that will divide into a group of numbers an equal number of times. The method for finding it is similar to L.C.M. except for the fact that we must find a divisor for each operation that will divide an even number of times into all of the numbers being considered.

For example: Find the G.C.D. of the numbers 132, 198, and 66.

It is evident that all of these numbers are divisible by 2. Therefore, we have

$$\begin{array}{r|l} 2 & 132 - 198 - 66 \\ \hline & 66 \quad 99 \quad 33 \end{array}$$

It is now clear that the numbers 66, 99, and 33 are all divisible by 11, so we have

$$\begin{array}{r|l} 11 & 66 - 99 - 33 \\ \hline & 6 - 9 \quad 3 \end{array}$$

The remaining numbers, namely, 6,9, and 3 are all divisible by 3, so our entire problem is set up thus:

$$\begin{array}{r|l} 2 & 132 - 198 - 66 \\ \hline 11 & 66 - 99 - 33 \\ \hline 3 & 6 - 9 - 3 \\ \hline & 2 - 3 - 1 \end{array}$$

Since we no longer find a number that will divide an even number of times into 2, 3, and 1, we stop at this point and multiply our divisors, 2, 11, and 3 giving us a product of 66 which is our greatest common divisor.

BASIC MATHEMATICS REVIEW

NAME_____ Lesson: IV

Worksheet: IV

DATE_____ Fractions and Denominators

IV. Fractions and Denominators

1. Find the least common denominators:

   (a) 5/6 + 1/9 + 2/3 =

   (b) 1/4 + 1/12 + 1/15 =

   (c) 1/50 + 1/100 + 1/25 =

   (d) 1/3 + 1/6 + 1/9 =

2. Addition of fractions: Match Column A with answers in Column B.

Column A		Column B
(a) 1/16 + 1/32 + 5/64 =	_____	(1) 1 23/28
(b) 5/6 + 1/9 + 3/4 =	_____	(2) 1 3/100
(c) 1/2 + 4/7 + 3/4 =	_____	(3) 1 5/16
(d) 7/8 + 1/4 + 3/16 =	_____	(4) 1 25/36
(e) 1/100 + 8/25 + 7/10 =	_____	(5) 11/64

INFORMATION SHEET Lesson: V

V. Reciprocals

The reciprocal of a number is 1 divided by the number.

For example: The reciprocal of 3 is 1/3.
The reciprocal of a fraction is the fraction inverted.
Thus, the reciprocal of 7/8 is 8/7 or 1-1/7.

For practice, show the reciprocals of

(a) 5/7 (b) 5/9 (c) 11/14 (d) $\frac{5}{8}, \frac{4}{7}, \frac{3}{4}$

Suppose it is stated that a certain value, which we shall call X, equals the sum of 1 over the reciprocals of several smaller values, which we shall say are 4, 5, and 6. This type of problem can be worked out arithmetically like this:

(a) 1 over 4, 5, and 6 is shown as $\frac{1}{4}, \frac{1}{5}$, and $\frac{1}{6}$

(b) The sum of these fractions is shown as $\frac{1}{4} + \frac{1}{5} + \frac{1}{6}$

(c) The reciprocal of this value is shown as $\frac{1}{\frac{1}{4} + \frac{1}{5} + \frac{1}{6}}$

Working this out:

$$X = \frac{1}{\frac{1}{4} + \frac{1}{5} + \frac{1}{6}} = \frac{1}{\frac{15}{60} + \frac{12}{60} + \frac{10}{60}} = \frac{1}{\frac{37}{60}} = \frac{1}{1} \div \frac{37}{60} =$$

$$\frac{1}{1} \times \frac{60}{37} = 1\ \frac{23}{37}$$

INFORMATION SHEET Lesson: VI

VI. Percentage

The efficiency of electrical machinery is expressed as a percent. The concept of percent is used throughout the fields of electricity to describe and compare electrical effects.

Percent is, briefly, the rate per hundred. For example, 27% means 27 out of every hundred. Thus 27 percent of one dollar, or 100 cents, equals $\frac{27}{100}$ of 100 = 27 cents.

There are numerous places in the electrical field where we can use our knowledge of percentage such as in the efficiency of motors, voltage regulators, etc.

Let us solve three kinds of percentage problems:

1. How much is 8% of 618?

   Solution: 8% = 8/100 or .08 therefore we multiply 618 x .08 = 49.44 which is our answer.

2. Twenty-six (26) is what percent of 104?

   Solution: Write as a fraction $\frac{26}{104}$ and divide.

   Carry out to two decimal places and we have .25
   Multiply by 100 and we have 25%

3. If 4% of a number is 226, what is the number?

   Solution: Express the % as a decimal (.04) and divide the given part of the number by the decimal.

   226 ÷ .04 = 5,650

INFORMATION SHEET Lesson: VII

VII. Powers and square roots

If we take the number 3 and multiply it by itself a certain number of times, for example 3 x 3 x 3 = 27, the answer, 27, is called a power of 3. The number 3 is called a base of the power 27.

If the base is used twice as a factor, the power is called the second power, or square, of the base. If the base is used three times as a factor, the power is called the third power, or cube of the base. The higher powers have no special names. They are simply designated as the fourth, fifth, eighth, etc., power of the base.

To indicate the number of times a base is to be used as a factor, a small number, known as an exponent is written to the right and above the base. Thus, 3^2 indicates that 3 is to be raised to the second power, 4^3 means four to the third power, 8^{10} means eight to the tenth power, etc.

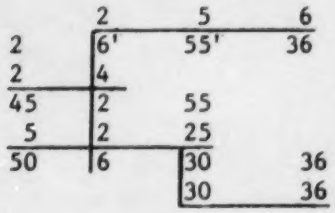
If we take the number 16 and separate it into two equal factors, we will find these factors must be 4. The number 4, then is called the square root of 16.

If we take the number 64 and separate it into three equal factors, we will find these factors must be 4. The number 4, then is called the cube root of 64.

The square root of a number, is one of two equal factors into which the number may be divided. The cube root of a number is one of three equal factors into which the number may be divided. The seventh root is one of seven equal factors, and so on for the higher roots.

To indicate that a root is to be taken, we use the radical sign $\sqrt{}$, and vinculum, _____, thus, $\sqrt{}$, placing a small figure, called the index in the opening of the radical sign to show which root is to be taken. The index ^{2}, to indicate square root, is omitted. Hence, a radical sign without an index number is understood to mean square root. For example, $\sqrt{16}$ means extract the square root of 16.

Let us extract the square root of a whole number, such as $\sqrt{65536}$



Let us proceed as follows:

1. Separate the number into periods of two each, beginning at the right. Since there are five numbers, this leaves tne first number (6) by itself.

2. The nearest square which is less than 6 is 2, so we place that number over the 6 and also to the left of it. Then we multiply 2 x 2 = 4 and place the 4 under the 6 and subtract, leaving 2 as our remainder.

3. Now, we bring down the next period of numbers which is 55.

5. Then we add the same number (2) to the two we already have at the left of the 6 and add. Thus, we have 4 leaving a blank space for our second number. Call it zero for the moment, if you wish.

5. Then, we divide 255 by 40. Since we are going to use our answer in place of the blank space or zero, we try 5 as our next number in the quotient, and place it over the pair of numbers (55).

6. Next we use this same number 5 in place of the zero or blank space to the right of the 4 and multiply. Thus, we have 5 x 45 = 225.

7. Now, we subtract 225 from 255 leaving 30 and bring down our next pair of numbers, namely 36.

8. Next we add the 5 with which we are now dealing to 45 which gives us 50, again leaving a blank space for our next divisor.

9. Since we now have four numbers (3036) divisible by three numbers (including the blank space) we can now say, 30 ÷ 5 equals 6 which we place in the quotient over the last pair of numbers (36). Then we place the same 6 in the blank space after 50 and multiply. Therefore, we have 506 x 6 = 3036 with no remainder. Our answer is 256, a perfect square, since 256 multiplied by itself will give us 65536 which is ample proof.

Let us now try a number containing a decimal, such as $\sqrt{456.6769}$

We now use our decimal point as the starting point from which we divide into periods of two each, counting to the left and also to the right. Thus, we have

		2	1	.	3	7	
2		4	56	.	67	69	
2		4					
41			56				
1			41				
423			15		67		
3			12		69		
426	7		2		98	69	
			2		98	69	

We proceed as before. Since 2 x 2 = 4 and subtracted from 4 leaves no remainder, we simply bring down the next two numbers (56) and divide by 40, leaving us 1 which we place over 56 (in our answer) and also in place of the zero in 40 and continue to multiply as previously shown.

NAME_____

DATE_____

Lesson: VII

Worksheet: VII

Square Roots

VII. Square Roots

1. Find the square roots of the following and check:

   (a) 625 = _____

   (b) 144 = _____

   (c) 10,000 = _____

   (d) 17.85 = _____

   (e) 4900 = _____

   (f) 18.288 = _____

INFORMATION SHEET Lesson: VIII

VIII. Ratio and Proportion

Ratio is the relation of one number to another of the same kind. Thus, the ratio of 8 to 12 is expressed as 8:12 or in the form of a fraction 8/12.

When it is required to determine what the relation of one number is to another, it is evident that the first is the dividend and the second is the divisor.

When it is required to determine the relation between two numbers, either may be regarded as dividend or divisor.

The first number is commonly regarded as the dividend.

The numbers compared are called the terms of a ratio. The first term is called the antecedent and the second term the consequent. The two together constitute a couplet.

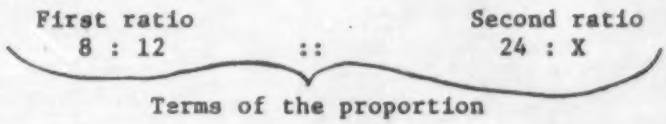
Ratio problems should be solved according to the following rules:

1. The terms of a ratio must be like numbers.

2. Multiplying or dividing both terms of a ratio by the same number does not change the ratio of the terms.

3. Fractions should be reduced to fractions having the same denominator, when reduced they will have the ratio of their numerators.

Proportion is an equality of ratios. In other words, when two ratios are equal the four terms form a proportion. A proportion is expressed by putting the sign = or :: between the ratios, thus:



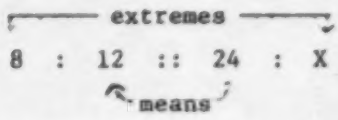
The proportion may also be written: $\frac{8}{12} = \frac{24}{X}$

The antecedents of the proportion are the first and third terms.

The consequents of the proportion are the second and fourth terms.

The first and fourth terms are called the extremes.

The second and third terms are called the means.



When solving problems in proportion, the following rules should be observed.

1. The product of the extremes is equal to the product of the means.
2. The product of the extremes divided by either mean gives the other mean.
3. The product of the means divided by either extreme gives the other extreme.

We shall deal with two kinds of proportion, the first known as direct proportion and the second as inverse proportion.

Direct proportion is one in which each term increases or diminishes, as the one on which it depends increases or diminishes.

For example: If an engine makes 475 revolutions in one minute, how many will it make in two minutes?

SMALL NO. OF REVOL.		LARGE NO. OF REVOL.		SHORT TIME		LONG TIME
475	:	X	::	1	:	2

Note that each ratio must contain quantities of the same kind. The unknown quantity is represented by the letter "X." Since, from the example it is evident that the consequent of the first ratio will be greater than the antecedent, write down the second ratio, so its consequent is greater than its antecedent, thus:

FIRST RATIO			SECOND RATIO		
REVOL.		REVOL.	TIME		TIME
475	:	X	1	:	2
(Small Term)		(Large Term) (unknown quantity)	(Small Term)		(Large Term)

Multiplying the means and extremes according to rule #1, we have X x 1 = 475 x 2 or 475 x 2 = 950 revolutions.

Lesson: VIII

NAME _____

Worksheet: VIII

DATE _____

Ratio and Proportions

VIII. Ratio and proportions

1. Find ratio of following quantities:
   Match Column B to Column A:

Column A		Column B
(a) 4 to 16 in.	_____	(1) 4 to 1
(b) 12 to 3 ft.	_____	(2) 4.5 to 1
(c) 9 ft. to 24 in.	_____	(3) 1 to 4

2. Solve the following direct proportions:

   (a) $\frac{20}{R} = \frac{5}{15}$

   (b) $\frac{6}{14} = \frac{E}{7}$

   (c) $\frac{E}{15} = \frac{30}{67}$

   (d) $\frac{6}{200} = \frac{P}{100}$

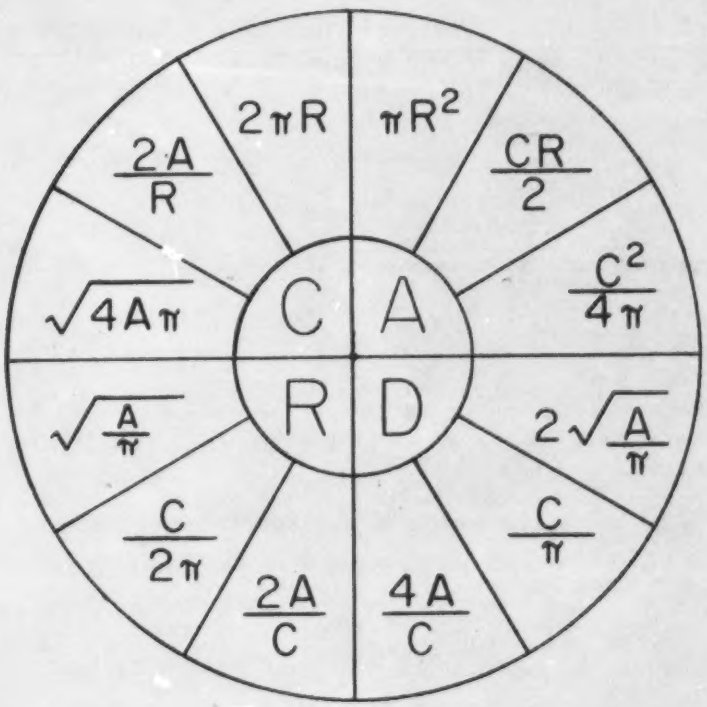
3. Write and solve proportions of the following:

   (a) A 1000 foot cable has a resistance of 30 ohms. How many ohms of resistance are in a cable 3000 feet long?

IX. The circle and the use of formulas

Since the circle becomes important in various phases of the electrical field, it is evident that we should bring to mind some of the more familiar formulas for finding the area, radius and diameter when certain problems are presented.

The following "wheel" will be of help. For instance, if you know the area or the radius, you can find the circumference by using one of the three formulas found opposite the letter "C" in the upper left-hand quadrant. If you know the radius or circumference of a circle, you can find the area by using any one of the formulas opposite "A" in the upper right-hand quadrant, etc.



Formulas

The first and most familiar one is $A = \pi R^2$ in which A = area; $\pi = 3.1416$; and R^2 = radius squared.

For example: Let us take the problem: Find the area of a circle whose radius is 5 inches. Since $A = \pi R^2$ we can substitute the known figures thus:

$A = 3.1416 \times (5)^2$ or $3.1416 \times 25 = 78.54$ square inches.

Now let us take a problem in which we know the area and desire to find the radius.
Find the radius of a circle whose area is 36 square inches.

Since: $\pi R^2 = A$, then $R^2 = \frac{A}{\pi}$ or $R = \sqrt{\frac{A}{\pi}}$ (See circle.)

$$R = \sqrt{\frac{36}{3.1416}} \quad \text{or} \quad R = \sqrt{11.4591} \quad \text{or 3.38 inches}$$

Now let us consider a problem in which we know the area, and desire to find the diameter.

For example: Find the diameter of a circle whose area is 29.75 square inches.

Since the diameter is twice that of the radius we can use the same formula as in the previous problem and multiply by 2 or we can use the formula:

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{29.75 \times 4}{3.1416}} = \sqrt{\frac{119}{3.1416}} = \sqrt{37.8787} = 6.15 \text{ inches.}$$

Using our radius formula and then multiplying by 2, we have:

$$R = \sqrt{\frac{29.75}{3.1416}} = \sqrt{9.47} = 3.075 \times 2 = 6.15 \text{ inches}$$

Rule: To eliminate the number which is multiplied by the unknown letter, divide both sides of the equals sign by the multiplier of the letter.

Example: Solve the following equations for the values of the unknown letters.

Solution:

1.	Write the equations.	$2R = 10$	$14 = 7E$
2.	Divide both sides of each equation by the multiplier of the letter.	$\frac{2R}{2} = \frac{10}{2}$	$\frac{14}{7} = \frac{7E}{7}$
3.	Cancel out the multiplier of the letters.	$R = \frac{10}{2}$	$\frac{14}{7} = E$
4.	Dividing.	$R = 5$ Ans.	$2 = E$ Ans.

Lesson: IX

NAME_____

Worksheet: IX

DATE_____ Formulas and Equations

IX. Formulas and Equations

1. What is the meaning of the following signs?

(a) (x)

(b) (-)

(c) (+)

(d) (-)

(e) (=)

(f) (.)

2. True or False

A formula is a rule or law, pertaining to relationship, expressed by means of letters, symbols and numbers, and used to solve an equation. _____

3. Solve the following equations:

(a) Formula E = IR

Find E if I = 10 and R = 12. E = _____

(b) Formula E = IR

If the value of E = 120 and the value of I = 8, would the value of R = _____. (Choose one) (1) 15 (2) 20 (3) 40 (4) 8

(c) Formula $R_t = R_1 + R_2 + R_3$

If $R_1 = 7$, $R_2 = 8$ and $R_3 = 5$, R_t would be _____.

(Choose one) (1) 23 (2) 21 (3) 20

4. Solve for the values of unknown letters:

(a) E - 7 x 10 =

(b) 10 = .2 x R =

(c) 600 = E x 50

(d) 120 = I x 1.5

INFORMATION SHEET Lesson: X

X. Negative exponents

It is common practice to indicate a decimal fraction consisting of many decimal places by the use of a negative exponent, the exponent being attached to the number 10 to indicate the number of figures to the right of the decimal point.

Thus: 0.00009 might be written 9×10^{-5}, $0.5000 = 5000 \times 10^{-4}$, $0.00000003 = 3 \times 10^{-8}$

Therefore: $6000 \times 10^{-12} = 0.000000006000 = 0.000000006 = 6 \times 10^{-9}$, $8 \times 10^{-7} = 0.0000008$, and $14 \times 10^{-5} = 0.00014$.

Let us proceed with a problem involving negative exponents such as the following:

$$\frac{10^{6} \times 10^{-4} \times 10^{-2} \times 10}{10^{-9} \times 10^{-3} \times 10^{14}}$$

The procedure for the solution of this type of problem is to combine the exponents as far as possible. Therefore, we may write the above problem as:

$$\frac{10^{6} \times 10^{-6} \times 10}{10^{-12} \times 10^{14}}$$

The next step is to move the negative exponents below the line to a position on top of the line, changing the minus sign to a positive sign as this is done.

The problem then becomes: $$\frac{10^{6} \times 10^{-6} \times 10^{12} \times 10}{10^{14}}$$

Next, add the exponents algebraically, writing the problem as:

$$\frac{10^{12} \times 10}{10^{14}} = \frac{10^{13}}{10^{14}}$$

Then cancel the zeros after the 10's as far as possible by subtracting 12 from each exponent. The problem then becomes:

$$\frac{10}{10^2} = \frac{10}{100} = \frac{1}{10} = 0.1$$

Now let us solve another similar problem:

$$\frac{0.004\times10^{6}\times2700\times10^{-7}}{0.00006\times10^{3}\times10^{-5}\times5\times2\times0.0002} = \frac{10.8\times10^{-1}}{24\times10^{-9}\times10^{-2}} = \frac{10.8\times10^{-1}\times10^{11}}{24}$$

$$\frac{10.8\times10^{10}}{24} = \frac{1.8\times10^{10}}{4} = \frac{18\times10^{9}}{4} = \frac{9\times10^{9}}{2} = 4.5\times10^{9} =$$

$45 \times 10^{8} = 4,500,000,000$